A First Course in Linear Algebra

Exercise 1. 20 Pag 113

Consider the homogeneous system of linear equations $\langle\text{homosystem}|A\rangle$, and suppose that $\langle\text{vect}|u\rangle = \langle\text{colvector}|u_1$
u_2
u_3
$\vdots$
$u_n\rangle$ is one solution to the system of equations. Prove that $\langle\text{vect}|v\rangle = \langle\text{colvector}|4u_1$
$4u_2$
$4u_3$
$\vdots$
$4u_n\rangle$ is also a solution to $\langle\text{homosystem}|A\rangle$.

Considere el Sistema Homogeneno de Ecuaciones Lineales (Sistema Homogeneo| A). y supone que (vect| u) = $\langle$colvector$|u_1$
u_2
u_3
$\vdots$
$u_n\rangle$ es una solucion del sistema de ecuaciones. Probar que (vect $|\boldsymbol{v}$) $=\langle$colvector$|4u_1$
$4u_2$
$4u_3$
$\vdots$
$4u_n\rangle$ es tambien una solucion para (sistema Homogeneo|A)

SOLUCION:

Suppose that a single equation from this system (the i-th one) has the form,

Suponga esa, una ecuacion simple para este sistema (the $\boldsymbol{i}$-th one) tiene la forma,

$$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \langle\text{dots}\rangle + a_{in}x_n = 0$$

Evaluate the left-hand side of this equation with the components of the proposed solution vector $\langle\text{vect}|v\rangle$,

Evaluando el lado izquierdo de esta equacion con los componenetes propuestos del vector solución$\langle\text{vect}|v\rangle$,

$$\begin{aligned}
a_{i1}(4u_1) &+ a_{i2}(4u_2) + a_{i3}(4u_3) + \langle\text{dots}\rangle + a_{in}(4u_n) \\
&= 4a_{i1}u_1 + 4a_{i2}u_2 + 4a_{i3}u_3 + \langle\text{dots}\rangle + 4a_{in}u_n && \text{Commutativity Conmutatividad} \\
&= 4(a_{i1}u_1 + a_{i2}u_2 + a_{i3}u_3 + \langle\text{dots}\rangle + a_{in}u_n) && \text{Distributivity Distributividad} \\
&= 4(0) && \langle\text{vect}|u\rangle \text{ solution to } \langle\text{homosystem}|A\rangle \\
&= 0 && (\text{vect}|u)\text{solucion para (Sistema Homogeneo}|A)
\end{aligned}$$

So $\langle\text{vect}|v\rangle$ makes each equation true, and so is a solution to the system.Notice that this result is not true if we change $\langle\text{homosystem} \mid A\rangle$ from a homogeneous system to a non-homogeneous system. Can you create an example of a (non-homogeneous) system with a solution $\langle\text{vect} \mid u\rangle$ such that $\langle\text{vect}|v\rangle$ is not a solution?

entonces (vect$|\boldsymbol{v}$) hace de esta ecuacion verdadera, y entonces es una solucion para el sistema. Note que este resultado no es cierto si cambiamos (sitema Homogeneo|A) de un sistema Homogeneo a Un Sistema No Homogeneo. Puedes crear un ejemplo de un sistema(No homogeneo)con una solucion (vect | $\boldsymbol{u}$) tal que (vect | $\boldsymbol{v}$) no es una solucion?